

11. Lines and angles

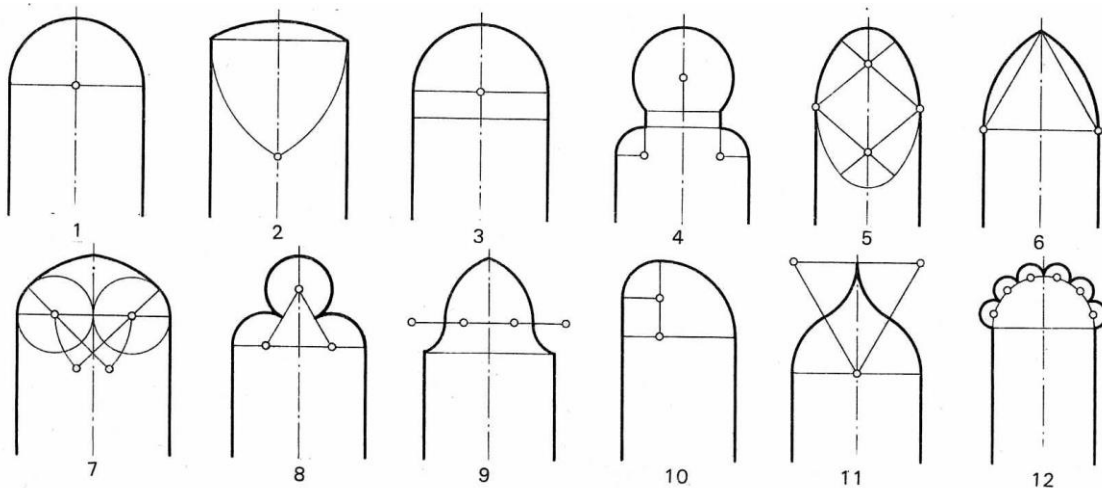
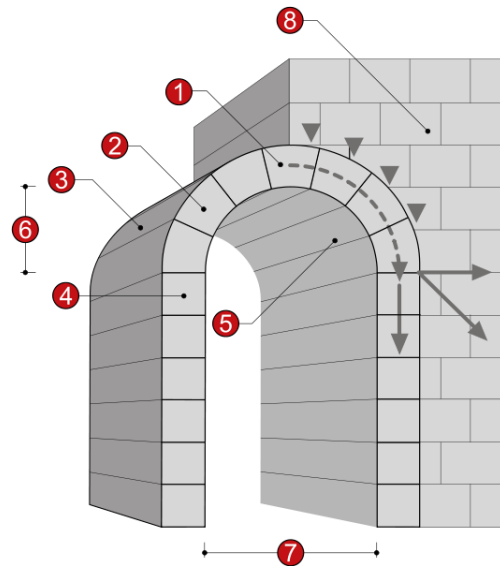
THE ARCH

An arch is a structure that creates space below and supports a load.

These are the parts of an arch:

1. Keystone (clave)
2. Voussoir (dovela)
3. Back (extradós)
4. Impost (imposta)
5. Intrados (intradós)
6. Rise (flecha)
7. Clear span, "Bay" (luz, vano)
8. Abutment (contrafuerte)

There are many different types of arches. Here are some of them and the way to build them.



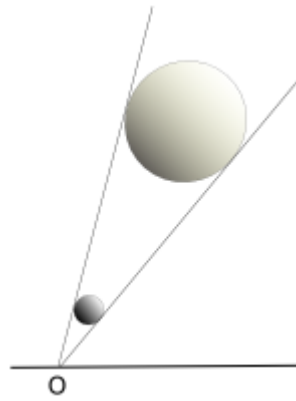
arcos: 1, de medio punto; 2, rebajado o escarzano; 3, peraltado; 4, de herradura; 5, elíptico; 6, apuntado u ojival; 7, carpanel apuntado; 8, trebolado; 9, escocés; 10, por tranquil o rampante; 11, conopial. 12, festoneado.

You have to find the way to make three of them and to explain step by step the process.

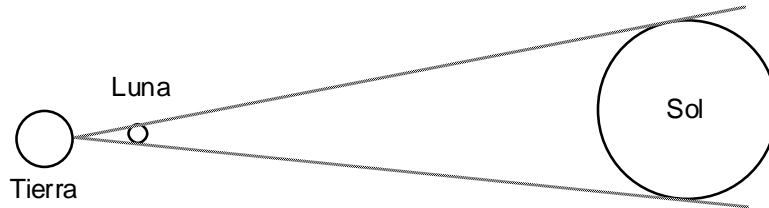
And finally you have to draw in your notebook with a ruler and a compass the chosen arches.

The starting point has to be the width and the height of the basic rectangle.

HOW TO MEASURE ANGLES



The apparent size of something is the angle that it represents from your point of view. That is the reason why it seems to be smaller when it is further from your position.



These are some easy ways to measure an angle by using your own body.

Fig.2

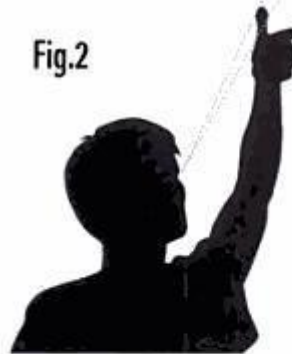
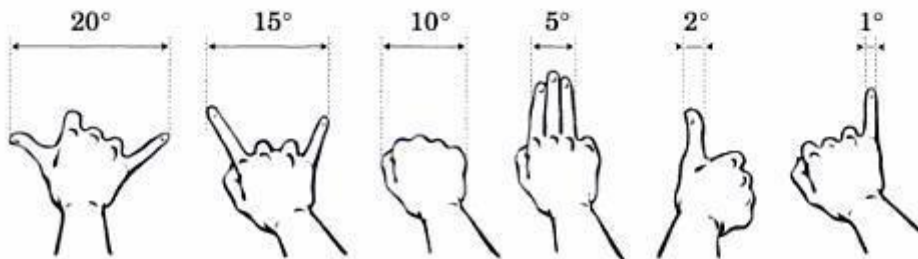


Fig.3



1. BASIC ELEMENTS OF THE PLANE

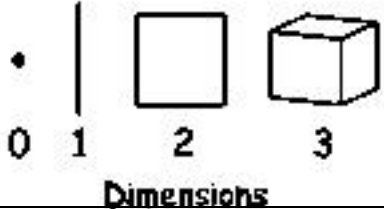
Geometry is the part of maths that studies:

- The space.
- The objects: their shapes and their properties.
- The transformations: translations, rotations, similarity, symmetry...

Plane geometry: is the geometry of two-dimensional space - the kind of space we have in a flat surface.

Solid geometry: is the geometry of three-dimensional space - the kind of space we live in.

BASIC ELEMENTS

<p>Point A point is the simplest object in the space. It has no size, only position.</p>	
<p>Lines A line is a continuous sequence of points. The unit of measurement is the metre. We have straight lines and curve lines.</p>	
<p>Angles An angle is the separation between two straight semi lines with a common origin point. The unit of measure angles results from the division of the circle into 360 parts. Each part is called a degree.</p>	
<p>Segment A segment is the part of a straight line between two fixed points.</p>	
<p>Straight semi line A semi line is one of the parts of a straight line divided by a point. It has an origin but no end.</p>	

<i>Secant lines</i> The lines which have a common point. They have different direction.	
<i>Parallel lines</i> The lines without any common point. They have the same direction.	

MESUREMENT OF ANGLES: SEXAGESIMAL SYSTEM

A sexagesimal degree is the amplitude of the angle obtained by dividing the circle into 360 equal parts.

A minute is the amplitude of the angle obtained by dividing a degree into 60 equal parts.

A second is the amplitude of the angle obtained by dividing a minute into 60 equal parts.

$$1^{\circ} = 60'$$

$$1' = 60''$$

$$1^{\circ} = 60 \times 60 = 3600''$$

And in the opposite way:

$$1'' = (1/60)'$$

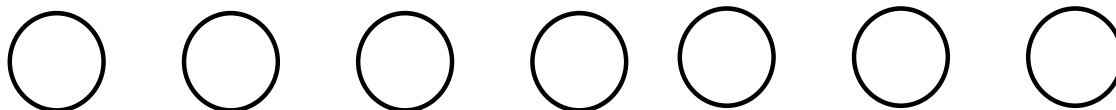
$$1' = (1/60)^{\circ}$$

$$1'' = (1/3600)^{\circ}$$

Example:

Draw without using a ruler, that is, approximately, the following angles into the circumferences:

a) 30° ; b) 45° ; c) 60° ; d) 75° ; e) 120° ; f) 180° ; g) 250°



2. OPERATIONS WITH ANGLES

We use a sexagesimal system to measure angles so we have to operate in this system.

Addition

We start by the seconds and so on.

We carry the excess to the next unit.

$$\begin{array}{r} 32^{\circ} \quad 24' \quad 48'' \\ + \quad 43^{\circ} \quad 49' \quad 25'' \\ \hline \end{array}$$

Subtraction

We start by the seconds and so on.

We convert a unit to the lower unit if we need to.

$$\begin{array}{r} 52^{\text{h}} \quad 23^{\text{m}} \quad 18^{\text{s}} \\ - \quad 43^{\text{h}} \quad 49^{\text{m}} \quad 25^{\text{s}} \\ \hline \end{array}$$

Multiplication of an angle by a number

We start by multiplying each unit by the number.

We carry the excess to the upper unit.

We start multiplying each unit by the number. We carry the excess to the upper unit.

$$\begin{array}{r} 27^{\circ} \quad 18' \quad 34'' \\ \times 4 \\ \hline 108^{\circ} \quad 72' \quad 136'' \\ \quad \swarrow \downarrow \\ \quad 2' \quad 16'' \\ \hline 108^{\circ} \quad 74' \quad 16'' \\ \quad \swarrow \downarrow \\ \quad 1^{\circ} \quad 14' \\ \hline \blacktriangleright 109^{\circ} \quad 14' \quad 16'' \end{array}$$

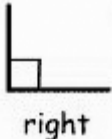
$$\begin{array}{r} 32^{\circ} \quad 23' \quad 49'' \\ \times \quad \quad \quad 5 \\ \hline \end{array}$$

Division of an angle by a number

We start dividing the degrees. We convert to a lower unit each remainder.

$$\begin{array}{r}
 66^{\circ} \quad 45' \quad 36'' \quad | \quad 4 \\
 2^{\circ} = \frac{120'}{165} \\
 1' = \frac{60''}{96''} \\
 0''
 \end{array}
 \qquad
 \begin{array}{r}
 37^{\circ} \quad 48' \quad 25'' \quad | \quad 5 \\
 16^{\circ} \quad 41' \quad 24''
 \end{array}$$

3. TYPES OF ANGLES

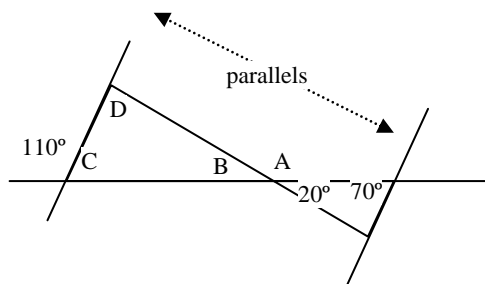
Right angle It is 90°		Convex angle It is larger than a zero angle and smaller than a straight angle.	
Straight angle It is 180°		Reflex angle It is between 180° and 360°	
Full rotation It is 360°		Complementary angles They add up to 90 degrees.	
Acute angle It is less than 90°		Supplementary angles They add up to 180 degrees.	
Obtuse angle It is between 90° and 180°			

4. EQUALITY OF ANGLES

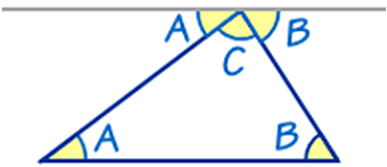
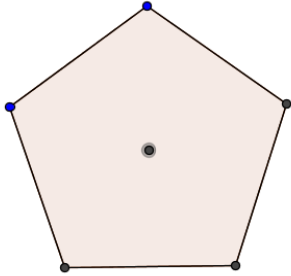
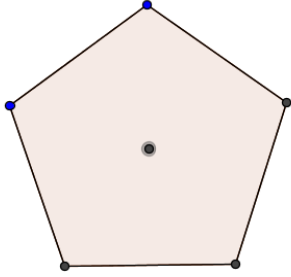
Vertical angles are equal.	
Corresponding angles formed by a common line and two parallel lines are equal.	
Corresponding angles formed by parallel lines are equal.	
Corresponding angles formed by perpendicular lines are equal.	

Example

Calculate every angle you see on the picture:

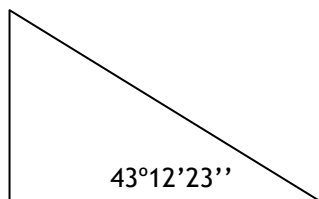


5. ANGLES IN A POLYGON

<p>Triangle</p> <p>In any triangle the angles always sum to 180°</p>	
<p>Central angle</p> <p>The angle formed by two consecutive radii. Its size is $CA = \frac{360}{n}$; n is the number of sides.</p>	
<p>Interior angles</p> <p>An interior angle is the angle formed by two consecutive sides. $IA = 180 - CA$</p>	

Example

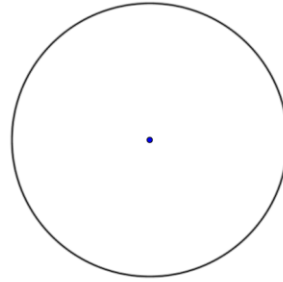
Calculate the unknown acute angle in this right triangle.



6. ANGLES IN A CIRCUMFERENCE

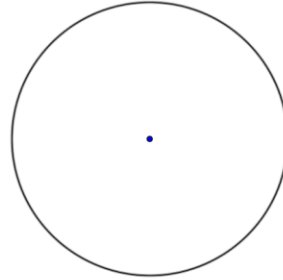
Central angle

Central angles are angles formed by any two radii in a circle.



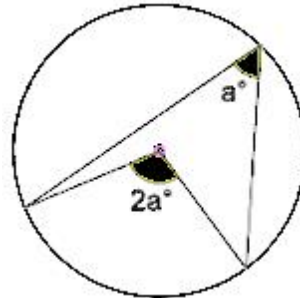
Inscribed angle

An angle made from points sitting on the circle's circumference.



Inscribed angle theorem

An inscribed angle a° is half of the central angle $2a^\circ$



EXERCISES AND PROBLEMS

1. Basic elements of the plane

1. What angle is formed by the hour hand and the minute hand of a clock at eight o'clock?
2. What angle is formed by the hands of a clock at half past twelve?
3. At half past three, what angle does the hands of a clock form?

2. Operations with angles

4. Work out the following operations of angles:
a) $56^{\circ}57'48'' + 27^{\circ}42'37'' + 23^{\circ}51'55''$
b) $135^{\circ}12'04'' - 43^{\circ}47'54''$
c) $(13^{\circ}42'34'') \cdot 7$
d) $134^{\circ}32'34'' : 12$
5. Do the following operations:
a. $127^{\circ} 25' 9'' - 123^{\circ} 57' 43''$
b. $7^{\circ} 46' 26'' \cdot 13$
6. Do the following operations:
a. $55^{\circ} 34' 28'' + 83^{\circ} 47' 52''$
b. $167^{\circ} 40' 18'' : 14$
7. Do the following operation: $84^{\circ}47'27'' : 5$
8. Do the following multiplication: $12^{\circ}48'35'' \cdot 8$
9. Do the following operations:
a) $57^{\circ}42'34'' + 13^{\circ}34'51''$; b) $130^{\circ}7'16'' - 90^{\circ}18'25''$
10. Do the following operations:
a) $5^{\circ}53'18'' \times 8$; b) $47^{\circ}32'51'' : 9$

3. Types of angles

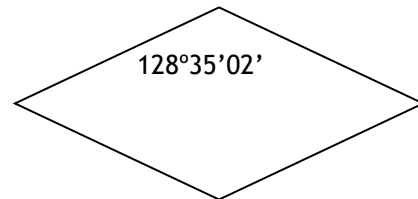
11. Determine the supplementary angle of $25^{\circ} 38' 40''$
12. Determine the complementary angle of $38^{\circ} 36' 43''$
13. Calculate the complementary angle to $23^{\circ}42'34''$
14. Estimate the supplementary angle to $123^{\circ}42'34''$
15. a) The unequal angle of an isosceles triangle is $45^{\circ}23'48''$. What are the values of the other angles?
b) Do the next operation: $84^{\circ}47'27'' : 5$
16. a) A rhombus has an angle of $53^{\circ}14'12''$. What are the others?
b) Do the operation: $12^{\circ}48'35'' \cdot 8$
17. Do the following operations:
a. $(55^{\circ} 34' 28'') \cdot 7$

b. Calculate the supplementary angle of: $103^{\circ} 52' 24''$

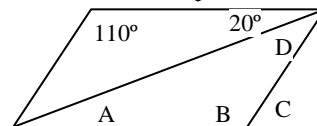
- 18.** Do the following operations:
a. Find the complementary angle of $27^{\circ} 25' 9''$
b. $47^{\circ} 46' 26'' : 13$

4. Equality of angles

- 19.** Calculate the unknown angles in this rhombus.



- 20.** Find and justify the value of angles A, B and C and D which are missing in the figure. The quadrilateral is a parallelogram.



- 21.** Find and justify the value of the angles A, B and C which are missing in the following figure:

